nite after body it is not clear how this could be applied to finite annular bodies. The numerical experimentation reveals that the distribution of panels should be such that the variation of source strengths should be gradual without any abrupt jumps. As mass flow ratio increases, the source strength decreases for the internal flow and increases for the external flow. Regarding panel distribution, sixty panels are used, with 30 panels on internal surface and 30 panels on external surface with one point at the leading edge. In the case of annular aerofoil with the central body, about 51 points are distributed on the aerofoil and 10 points on the central body. Even though the number of panels is very much less than what Smith has suggested,[1] the comparison is fairly good. The agreement near the trailing edge is poor. The reason may be due to the interference effects of the electrodes kept near the trailing edge for controlling the mass flow.

Conclusion

From the above discussion it is clear that the rheoelectric analogy is a good first approximation in solving the flow problems past annular aerofoils with and without central bodies. The main advantage in its use is the simplicity, and the continuous representation of the flowfield without involving any discretisation. Comparison with Young's nonlinear theory is fairly good. Smith's method using source distributions gives a good representation of flow past thick semi-infinite annular aerofoils without controlling mass flow, as shown in Fig. 3.

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Roll-Up of Aircraft Trailing Vortices using Artificial Viscosity

A. M. Bloom* and H. Jen†

Joint Institute for Acoustics and Flight Sciences, NASA Langley Research Center, Hampton, Va.

Nomenclature

b = span
c = chord
C_L = lift coefficient
r = radial coordinate
U = freestream velocity
u = axial velocity component
v = spanwise velocity component
w = vertical velocity component
x = streamwise coordinate
y = spanwise coordinate
z = vertical coordinate
Γ = circulation
κ = trailing vortex strength
ν = kinematic viscosity
$\bar{\nu}$ = $\nu/(Uc)$

Subscripts

$(\ )_c$ = vortex center
$(\ )_f$ = flap

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Index category: Aircraft Aerodynamics (Including Component Aerodynamics).

*Assistant Research Professor; also Assistant Research Professor of Engineering, School of Engineering and Applied Science, George Washington University, Washington, D.C. Member AIAA.

†Research Assistant.

Introduction

IN the past several years considerable interest has been shown in computational methods of analyzing the roll-up characteristics of aircraft trailing vortices. Although the general problem is quite complicated, considerable progress has been made by assuming a potential flow model. This idea was first introduced by Westwater.[1] Since that time, various applications and extensions of the Westwater type of method have been made.[2-5] Examination of the computational results of these investigations reveals that all of these authors encountered an irregular roll-up of vortex sheet in the tip region.

In a recent study of the numerical representation of vortex sheets Kuwahara and Takami[6] found that by including an "artificial viscous core" term in the equation for induced velocity with a suitably chosen value of the "artificial viscosity coefficient" the irregularities that normally occur in the Westwater method can be smoothed out. (A similar approach also has been proposed by Chorin and Bernard.[7])

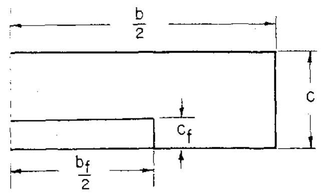
The purpose of the present investigation was to apply the artificial viscosity method of Kuwahara and Takami to a number of practical aerodynamic configurations and compare the results for the core location with existing experimental data where possible.

Mathematical Model

Following Kuwahara and Takami the induced velocity field of a collection of two dimensional line vortices with artificial viscosity distributed over the span of the wing is given by

$$u_i = \frac{dx_i}{dt} = U \text{ or } x_i = t_i$$

$$v_i = \frac{dy_i}{dt} = -\frac{1}{2\pi}\sum_{j\neq i}\kappa_i\left[1 - \exp\left(\frac{-r_{ij}^2}{4\nu t}\right)\right]\frac{(z_i - z_j)}{r_{ij}^2}$$



Configuration Number	Description	C_L	c_f/c	δ_f	b_f/b	b	c
1	Plain Wing	1.0	-	-	-	8	1
2	Flapped Wing	1.5	0.25	20	0.5	8	1
3	Flapped Wing	1.0	0.25	40	0.5	8	1

Fig. 1 Wing-flap geometry and aerodynamic configurations.

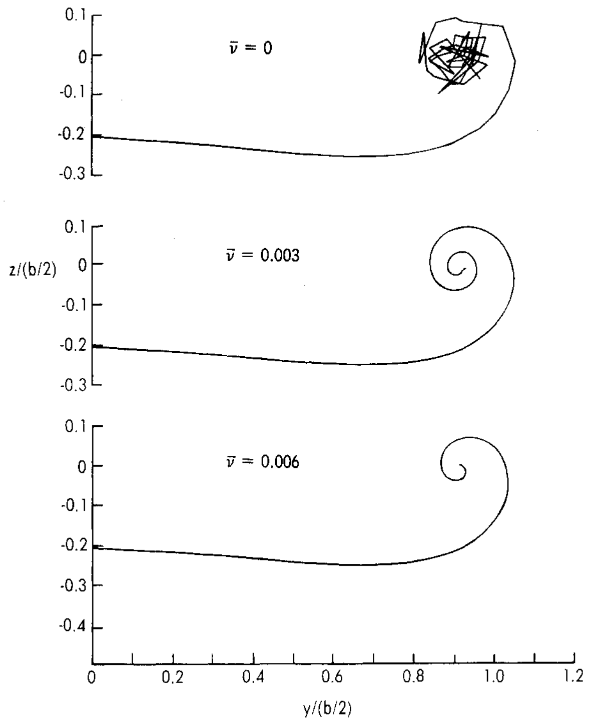


Fig. 2 Configuration 1 with $\bar{\nu}$ = 0, 0.003, and 0.006, X/C = 15.

$$w_i = \frac{dz_i}{dt} = \frac{1}{2\pi}\sum_{j\neq i}\kappa_i\left[1 - \exp\left(\frac{-r_{ij}^2}{4\nu t}\right)\right]\frac{(y_i - y_j)}{r_{ij}^2}$$
$$i,j = 1,2,\ldots,N$$

where

$$r_{ij}^2 = (y_i - y_j)^2 + (z_i - z_j)^2$$

The computer program of Ref. 8 was used to calculate the discrete spanwise circulation distribution for a given wing-flap configuration and lift coefficient.

Numerical Results

The aerodynamic configurations and planform geometry considered in the present investigation are shown in Fig. 1. Figure 2 shows the results for configuration 1 at 15 chords downstream for $\bar{\nu}$ values of 0, 0.003, and 0.006. It is clear that $\bar{\nu}$ = 0.003 is sufficient to smooth out the irregularities in the tip region. Increasing $\bar{\nu}$ to 0.006 merely delays the roll-up process. The value of $\bar{\nu}$ = 0.003 was used for all additional calculations. Figure 3 shows the results for configurations 2 and 3 at 15 chords downstream. The roll-up process for configuration 2 is about equally divided between the flap and wing-tip vortex whereas configuration 3 has a dominant flap vortex.

Comparison with Experimental Data

Chigier and Corsiglia[9] have experimentally determined the position of the vortex centerline using a three component hot wire probe behind a semispan rectangular wing with a NACA 0015 airfoil section. Figure 4 shows a comparison of experimental vortex centerline positions with the theoretical estimate as determined by the method presented in this paper. The theoretical vortex center is obtained from the center of curvature of the vortex sheet in the tip section. At $x/c = 0$ the theory assumes that the roll-up begins at $y_c/b = 0$ while the data reflects initial roll-up of the flow at approximately the quarter chord of the wing. For $x/c > 0$ the theory begins to pick up the trend of the data, especially for y_c/b. For z_c/b the theory shows an increasing deviation from the data. It is believed that part of this deviation is due to the influence of the wind tunnel walls on the vortex location. If this is the case, Ref. 10 indicates that a boundary induced upwash is present in the data. The solid data points in Fig. 4 represent the data of Ref. 9 corrected for the upwash using the method of Ref. 10. The theoretical prediction is found to lie between the uncorrected and corrected data.

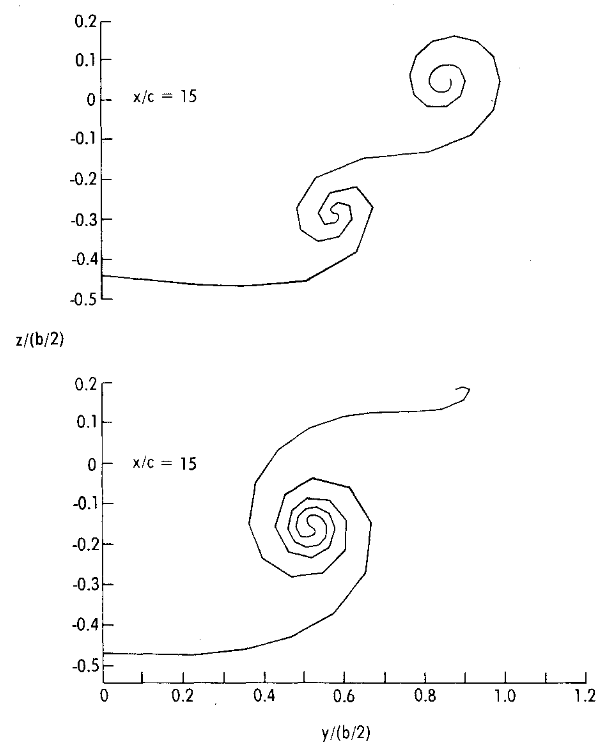


Fig. 3 Configurations 2 and 3 with $\bar{\nu}$ = 0.003.

Conclusions

An artificial viscosity method has been used to calculate the roll-up of trailing vortices behind a number of practical aerodynamic configurations. Regular roll-ups

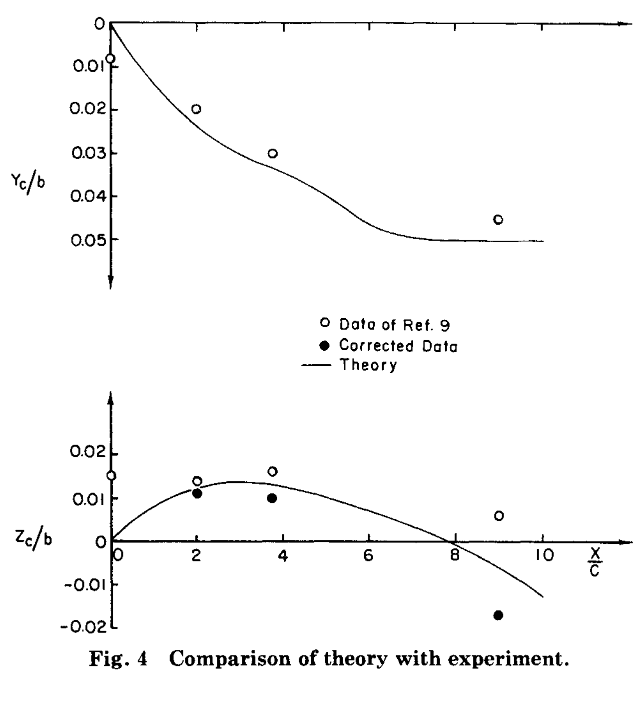


Fig. 4 Comparison of theory with experiment.

were obtained even for the case of partial span flaps. The theoretical estimate of the vortex core location compared reasonably well with the spanwise position measurement. The vertical position measurement may be significantly influenced by the wind tunnel walls.

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Correlation for Estimating Vortex Rotational Velocity Downstream Dependence

D. L. Ciffone*

NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

AR = aspect ratio
b = wing span
C_L = lift coefficient
$f(\Gamma_0/\nu)$ = Reynolds number parameter
R_{e_b} = Reynolds number based on wing span
U_∞ = freestream velocity (towing speed for water tank)
V_1 = vortex maximum rotational velocity
X = downstream distance, aft from trailing edge of wingtip
α = angle of attack
Γ_0 = circulation shed from one side of wing
Γ_0/ν = vortex Reynolds number

Subscripts

B = downstream distance where plateau region ends
P = plateau region

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Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Research Scientist.

Introduction

LIFT-GENERATED wake vortices trailing behind present-day heavy aircraft are currently of great concern to the aviation community. In order to minimize the upset potential which these vortices present to following aircraft, it is necessary to have an understanding of the wake characteristics in the far field, where such encounters are likely to occur.

Available wind-tunnel data of wake vortex velocity profiles are limited by the physical lengths of tunnel test sections. Downstream distances of up to 30-span lengths have been obtained in the NASA Ames 40- by 80-ft Wind Tunnel.[1] However, these distances are not great enough to obtain the characteristics of vortex decay. Recent measurements[2,3] made in the wake of wings being towed under water have identified two characteristic flow regions for the dependence of vortex maximum tangential velocity on downstream distance. The first, a "plateau" region, with little, if any, change in maximum tangential velocity, extends from wake rollup to downstream distances as great as 100-span lengths, depending on span loading and angle of attack. This is followed by a decay region in which the maximum tangential velocity decreases with downstream distance at a rate nominally proportional to the inverse one-half power. This note describes a correlation of these water-tank results that relates the magnitude of vortex maximum tangential velocity in the plateau region to the downstream extent of this region. With this knowledge, near field wake velocity measurements in the plateau region (i.e., wind-tunnel results) can be used to estimate the far field vortex characteristics.

Plateau Region Correlation

A correlation function based on the self-similar turbulent decay of a line vortex has been developed[4] and utilized to substantiate the validity of using ground-based scale model data to predict high Reynolds number flight results. It was found that if the downstream dependence of vortex maximum rotational velocity is presented as a vortex velocity scaling parameter, $V_1 b/\Gamma_0 AR$, vs a distance scaling parameter, $(X/b)(\Gamma_0/U_{\infty b})(AR)^2 f(\Gamma_0/\nu)$, the scale model and flight data collapse to a single curve. Presented in Fig. 1 are the results of employing these scaling parameters to plot vortex maximum rotational velocity in the plateau region versus a corresponding downstream duration of this region obtained from the experimental data of Refs. 2 and 3. The line faired through the experimental points has a slope of -1. This implies the following in-

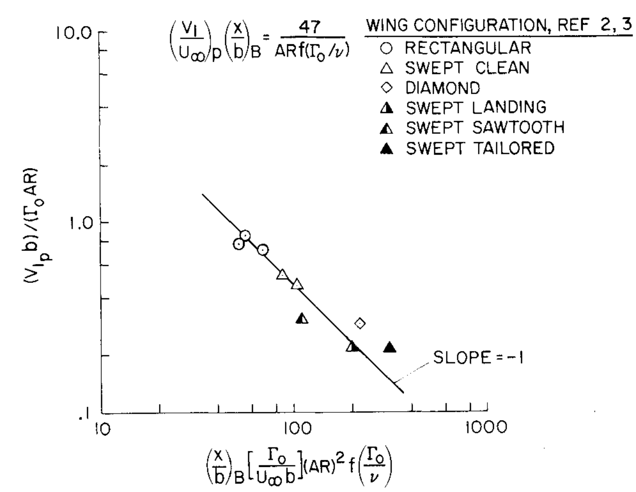

Fig. 1 Correlation of vortex maximum rotational velocity in plateau region to downstream duration of this region.